Journal of
Mechanical
Science and
Technology

Journal of Mechanical Science and Technology 21 (2007) 536~548

A Review on the Analysis and Experiment of Fluid Flow and Mixing in Micro-Channels

Simon Jayaraj[b], Sangmo Kang[a], Yong Kweon Suh[a,*]

[a]*Department of Mechanical Engineering, Dong-A University*
840 Hadan-dong, Saha-gu, Busan 604-714, Korea
[b]*On sabbatical leave from National Institute of Technology, Calicut, India*

(Manuscript Received August 14, 2006; Revised January 4, 2007; Accepted January 31, 2007)

Abstract

The studies with respect to micro-channels and micro-mixers are expanding in many dimensions. Most significant area of micro-mixer study is the flow analysis in various micro-channel configurations. The flow phenomena in micro-channel devices are quite different from that of the macro-scale devices. An attempt is made here to review the important recent literature available in the area of micro-channel flow analysis and mixing. The topics covered include the physics of flow in micro-channels and integrated simulation of the micro-channel flow. Also, the flow control models and electro-kinetically driven micro-channel flows are dealt in detail. A survey of important numerical methods, which are currently popular for micro-channel flow analysis, is carried out. Different options for mixing in micro-channels are provided, in sufficient detail.

Keywords: Micro-channel, Mixing, Flow Analysis, Review

1. Introduction

The phenomenon of mixing is so fundamental, that we face it everywhere in our daily life. The applications of mixing range from chemical reactions in industries to environmental studies with respect to pollution. The importance of mixing in micro-scale applications can never be under-estimated. Especially, the successful operation of new micro-devices, such as the lab-on-a-chip or μTAS, very much depends on the efficient mixing process of various fluid constituents.

Unlike the macro-scale applications, the micro-channels provide new challenges for carrying out the mixing operation effectively, the foremost reason being the low Reynolds number value of the micro-channel flow. Also, the small and intricate dimensions of the flow-conduit in micro-channels do not easily permit any mechanical stirring arrangement, which can help to enhance the mixing.

By definition, micro-channels are flow domains having their internal dimensions within the range of 1mm and 1μm. Due to various advantages that micro-channels offer, their applications range from compact heat exchangers to MEMS devices that are used for biological and chemical analysis. At present, micro-channels are used to transport and mix biological materials such as proteins, DNA, cells, etc. or to send chemical samples from one place to the other. A comprehensive coverage about micro-channel applications and the important aspects of the micro-channel flow are provided by Gad-el-Hak (2006).

This paper presents a review on the micro-channel flow and mixing. A highlight about modeling of micro-channel flows is elaborated in Sec. 2. Different

*Corresponding author. Tel.: +82 51 200 7648; Fax.: +82 51 200 7656
E-mail address: yksuh@dau.ac.kr

channel configurations and fluid models are discussed here. A note on the experiments with micro-channels is given in Sec. 3. In the micro-scales, fluid stirring becomes one of the most crucial factors in the design and operation of micro-devices. Explanations about the electro-kinetic transport and mixing in micro-channels are presented in Sec. 4 and Sec. 5, respectively.

2. Modeling Micro-Channel Flow

The physics of flow and energy transfer in micro-channel devices is found to be quite different from that of macro-scale devices. A well known example for this is the failure of Fourier law to predict the thermal conductivity of micro-structures (Gen, 2001). Luckily, the potential interaction distance is very small in typical liquid systems and the continuum description is valid for most micro systems, as well. Fluid flow in micro-channels typically requires higher pressure differences and the flow rates are very small. Thus micro-channel flow phenomena often demand a different way of analysis and special numerical tools.

The understanding of the unconventional physics involved in the manufacture and operation of small devices is crucial for designing, optimizing, fabricating and utilizing the improved MEMS. In dealing with the flow through micro-devices, the selection of physical and mathematical models, boundary conditions and solution procedure are quite critical. It is well known that the surface effects are going to dominate in micro-devices (Knight, 1999). The million-fold increase of surface area relative to the mass of the minute devices (per unit volume), substantially affects the transport of mass, momentum and energy through the surface.

A major difficulty in connection with the micro-channel analysis is the geometrical complexity of the flow domain. Numerous designs of micro-channels have been proposed, including T and H shaped channels, zigzag shaped channels, 2D and 3D serpentine channels and multi-laminators. The T-sensor designed by Weigle and Yager (1999) for implementation of assays in micro-channels is quite popular. In this design a reference stream, a detection stream and a sample stream are introduced through multiple T junctions into a common channel. Differential diffusion rates are also fundamental to the design of H-filter, used to separate components (Sculte et al., 2000). Splitting the streams and re-layering increases the interfacial area which promotes mixing. This layering approach was implemented by Branebjerg et al. (1996). By adding complexity to the flow field, this has good potential to increase the amount of mixing between the streams. The three dimensional serpentine channel proposed by Liu et al. (2000) was designed to introduce chaotic advection into the system and further enhance mixing.

2.1 Integrated simulation of micro-channel flow

For a larger class of flows, Navier-Stokes equations based on the continuum assumption are adequate to model the fluid behavior. Continuum approximation implies that the mean free path of the molecules, λ, in a gas is much smaller than the characteristic length, L. That is, the Knudsen number, $Kn = \lambda / L$, has to be very small (<<1). However, there are special cases in micro-channel flow, that do not satisfy this condition. The various flow regimes and suitable fluid models are shown in Table 1 (Arkilic et al., 1997).

As shown in this table, Navier-Stokes equation together with Boltzmann equation describes the flow in all regimes. For the accurate prediction of the flow parameters in micro-channels, it is very important that a suitable numerical tool is used for the analysis. Often in the literature it is found that the standard Navier-Stokes solvers are extended to yield solution for micro-cha-nnel flow parameters. While doing so, one should be very careful to incorporate the additional consider-ations with respect to the micro-channels. Along with the Navier-Stokes solvers,

Table 1. Flow regimes & fluid models for micro-channels.

Knudsen Number	Fluid Model
$Kn \to 0$ (continuum, no molecular diffusion)	Euler equations with slip-boundary conditions
$Kn \le 10^{-3}$ (continuum, with molecular diffusion)	Navier-stokes equations with no-slip-boundary conditions
$10^{-3} \le Kn \le 10^{-1}$ (continuum-transition) conditions	Navier-stokes equations with slip-boundary
$10^{-1} \le Kn \le 10$ (transition)	Burnett equations with slip-boundary conditions Moment equations DSMC Lattice Boltzmann
$Kn > 10$ (free molecular flow)	Collisionless Boltzmann DSMC

molecular based numerical simulation methods (for example, the lattice Boltz-mann method) for liquid and gas micro-flows are also being tried.

Many diverse processes with disparate spatial and temporal scales are involved in the full-system simulation of micro-channels. Fluidic devices often require full simulation of the individual components. This is particularly true for new designs, for which the lumped models and continuum approximations are inappropriate. To develop a system level simulation that is sufficiently accurate and robust, all processes involved must be simulated at a comparable degree of accuracy and integrated seamlessly.

Liquid and gas flows in micro-devices are characterized by low Reynolds number and unsteady nature. The Reynolds number values are typically of order one or less in channels with heights in the sub-millimeter range (Ho and Tai, 1998). The unsteadiness is due to the external excitation from a moving boundary or an electric field, often driven by high-frequency oscillators (Freeman et al., 1998). The domain is largely three dimensional and geometrically complex, consisting of large aspect ratio components, abrupt expansions and rough boundaries. In addition, micro-devices interact with larger devices, resulting in fluid flow going through disparate regimes. Accurate and efficient simulation of micro-flows should take into account the above factors. For example, the significant geometric complexity of MEMS flows suggests that finite elements and finite volumes are more suitable than finite differences for efficient discretization (Ye et al., 1999). A particularly promising approach applied recently for MEMS flow analysis makes use of mesh-less and mess-free approaches as explained by Aluru (1999).

In coupled domain problems, such as flow-structure or structure-electric (or a combination of both), there are significant disparities in temporal and spatial scales. These disparities, in turn, imply that multiple grids and heterogeneous time-stepping algorithms may be needed for discretization, leading to very complicated (and consequently), computationally prohibitive simulation algorithms. Possible construction of low-order dynamical models could be achieved by projecting the results of detailed numerical simulations onto spaces spanned by a very small number of degrees of freedom; the so-called non-linear macro-modeling approach (Senturia et al., 1997).
Construction of macro-models is not always possible, and this lack of simplified models for the many and diverse components of micro-systems makes system level simulation a challenging task. As there is enough progress in model development of electronic components, now considerable attention needs to be focused on models for non-electronic components. An illustration on the full simulation of fluidic system, which involves interaction with adjacent structures, is presented in Fig. 1. This is an anemometer-type flow sensor, which is made up of a heating element and two sensing elements (Ras-mussen and Zaghloul, 1999). The temperature dif-ference between the sensors is used to measure the flow parameters.

2.2 Molecular based micro-fluidic simulation

Since micro-channels have very small volume-to-surface ratios, the surface forces are more dominant

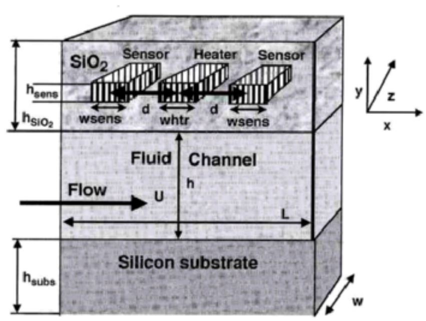

Fig. 1. Structure of an anemometer type flow sensor (Ras-mussen and Zaghloul, 1999).

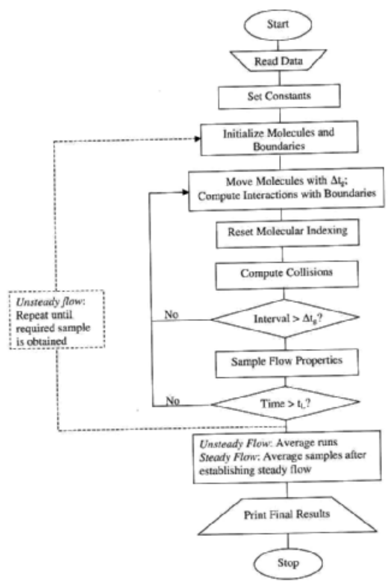

Fig. 2. Steps for a typical DSMC method.

than the body forces in such small scales. The origin of the surface forces is atomistic and based on the short-ranged van der Waals forces and longer ranged electrostatic or Coulombic forces. Although molecular simulation based approach for understanding fluid forces on surfaces is fundamental in nature, it is very difficult to apply the same to engineering problems, due to the vast number of molecules involved in the analysis. Also, the fundamental simulation approaches, from a microscopic point of view, differ for liquid and gas flows. Examples for molecular based micro-scale transport modeling are the Direct Simulation Monte Carlo (DSMC) algorithm and the Lattice Boltzmann Method (LBM).

The DSMC method was invented by Bird (1994) and several review articles are currently available (for example, Oran et al., 1998). This method is based on splitting the molecular motion and inter-molecular collisions by choosing a time step, less than the mean collision time and evolution of this molecular process in space and time. For efficient numerical implementation, the space is divided into cells (which are proportional to the mean free path, λ) similar to the finite-volume method. The basic steps of the DSMC algorithm, as explained by Oran et al. (1998), is shown in Fig. 2. It should be noted that, irrespective of its attractions, there are several possible limitations and error sources within the DSMC method.

Micro-scale thermal/fluidic transport in the entire Knudsen regime ($0 \le Kn < \infty$) is governed by the Boltzmann equation (BE). This equation describes the evolution of a velocity distribution function by molecular transport and binary intermolecular collisions; which is given in the following form (Bird, 1994) for the case of a simple dilute gas domain:

$$\frac{\partial nf}{\partial t} + c\frac{\partial nf}{\partial x} + F\frac{\partial nf}{\partial c} = \int_{-\infty}^{\infty}\int_{0}^{4\pi} n^2 (f^* f_1^* - f f_1)\, c_r\, \sigma\, d\Omega\, dc_1$$

where f is the velocity distribution function, n the number density, c the molecular velocity, F the external force per unit mass, c_r the relative speed of class c molecules with respect to class c_1 molecules, and σ the differential collision cross section. The Lattice Boltzmann method is based on the solution of this equation on a previously defined lattice structure with simplistic molecular collision rules. A good review article on LBM is given by Chen and Doolen (1998). The LBM can be viewed as a special finite differencing scheme for the kinetic equation of the discrete velocity distribution function, and it is possible to recover the Navier-Stokes equations from the discrete lattice Boltzmann equation with sufficient lattice symmetry (Frish et al., 1986).

During the last decade, LBM has been developed as an alternative and versatile numerical simulation method in computational fluid dynamics (CFD). LBM has relatively simple algorithms, and it has the potential for obtaining an easier solution for fluid flow problems instead of solving the Navier-Stokes equations (Qian et al., 1992). A flow chart giving step by step procedure of the free energy model in LBM is shown in Fig. 3. Many complicated CFD problems (multi-phase flows, multi-component flows, turbulent flows, etc.) have been simulated by researchers in the recent past, using LBM. It is easier to apply LBM to fluid applications involving interfacial dynamics and complex boundaries (Sukop and Thorne, 2006). Another useful application of the LBM is for granular flows, which can be expanded to include flow through micro-filtering systems (Angelopoulos et al., 1998). An interesting numerical simulation of two-phase flow within a micro-channel using free energy model of the LBM is carried out recently by Li and Suh (2006). Of course, LBM is not free of disadvantages. Firstly, implementation of boundary conditions

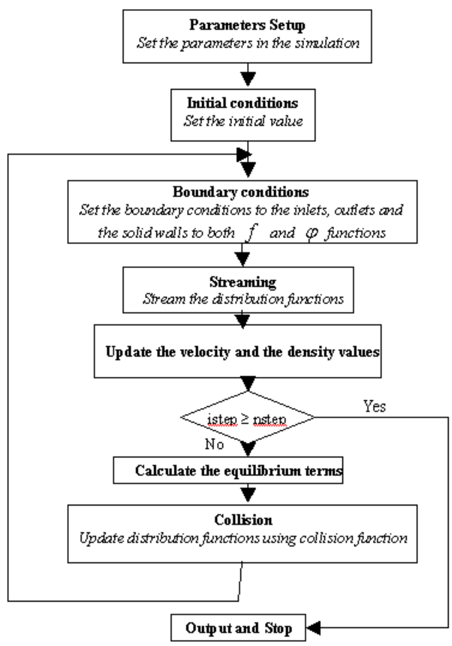

Fig. 3. Free-energy model based LBM algorithm.

is not straightforward as in the Navier-Stokes solvers, because the primary variable is the distribution function not the velocity field itself. Secondly, since LBM is basically based on the compressible-fluid assumption, the solutions contain the compressible effect anyhow, and so when the exact incompressibility should be a very important restriction, LBM is not suitable. These are currently the main issues among the researchers involved in the development of LBM techniques.

3. Experiments with micro-channel flows

During the last decade and more, numerous reports have appeared in the literature about experiments with micro- channels. The experimental study of micro-channel flow is plagued by many discrepancies, owing to the difficulty in conducting micro-scale measurements. Despite the fundamental simplicity of laminar flow in straight ducts, it often failed to reveal the expected relationship between the friction factor and Reynolds number with respect to micro-channels. Table 2 gives a summary of some experiments conucted by various researchers to investigate the behavior of fluid flow over a large range of Reynolds numbers, geometries and experimental conditions. Most of these experiments are aimed at determining the friction factor with respect to micro-channels. The inconsistencies observed by these experiments demonstrate the need for detailed velocity measurements in order to elucidate potential micro-scale effects and mechanisms in these channels.

3.1 Velocity measurement in micro-channels

Measurement of velocity in micro-channels has been carried out using bulk flow, point-wise or field measurements. Each of these techniques has certain advantages making it more suitable to provide a specific type of flow-field information. The majority of flow resistance data to date has been obtained through the use of bulk flow measurements. It is the most simple and cheap method of flow measurement without any optical access and seeding. The point-wise and field velocity measurements were essentially done by micro-PIV (Particle Image Velocimetry), first made by Santiago et al. (1998).

Alternate visual methods applied to micro-channel velocity measurements have been demonstrated by numerous researchers (for example, Brody et al., 1996). Molecular Tagging Velocimetry (MTV) was adapted to micro-scale by Webb and Maynes (1999), where velocity profiles were obtained in circular tubes with d =705 μm and Re=800 to 2200. Micro-PIV and Nuclear Magnetic Resonance (NMR) techniques have also been applied to the measurement of velocities in electro-osmotic flow (Cummings et al., 1999). An adaptation of Laser Doppler Anemometry

Table 2. Micro-channel experiments.

Channel Description		Reynolds Number Range	Reference
Shape	Size		
Trapezoidal/ Rectangular	d =50~56 μm w =287~320 μm l =1cm	200~600	Tuckerman & Pease (1981)
Trapezoidal or U-shaped	d =28~65 μm w =40-150μm l =7.6-40.3mm	200~ 15,000	Wu & Little (1983)
Rectangular/ Trapezoidal	d =0.48~38.7 μm w =55~115 μm l =10.2~10.9 mm	<<1-80 (approx.)	Pfahler et al. (1991)
Rectangular	d =100~300 μm w =200~400 μm l =50 mm	50~ 4,000	Peng et al. (1994)
Trapezoidal	d =20~40 μm w =40~150 μm l =11.7 mm	17~126	Wilding et al (1994)
Circular Trapezoidal & Triangular	d =8~42 μm d =13.4~46 μm w =35~110 μm l =2.5~10 mm	<1.2 (approx.)	Jiang et al. (1995)
Trapezoidal	d =27~63 μm w =100~1000 μm l =12~36 mm	<600	Flockhart & Dhariwal (1998)
Rectangular (channel array)	d =30 μm w =600 μm l =3 mm	1~18	Papautsky et al. (1999a)
Rectangular (channel array)	d =22.71~26.35 μm w =150~600 μm l =7.75 mm	0.001~10	Papautsky et al. (1999b)
Trapezoidal	d =28~114 μm w =148~523 μm l =28 mm	10~1,450	Qu et al. (2000)
Circular	d =3~81 μm	30~ 20,000	Choi et al. (1991)
Circular	d =19~102 μm	250~ 20,000	Yu et al. (1995)
Circular	d =50~254 μm	upto 2,500	Mala & Li (1999)
Circular	d =75~242 μm	50~2,500	Sharp et al. (2000)

d =depth/diameter; w =width; l =length

(LDA) techniques to micro-scale flows was demonstrated by Tieu et al. (1995), with point-wise data obtained in a 175 μm channel.

4. Electro-kinetics in micro-fluidics

Another important aspect which has to be taken care of, in the case of micro-channel flow and mixing analysis, is the electro-kinetic effect. Electro-kinetics is a general term describing the phenomena, which involve interaction between solid surfaces, ionic solutions and macroscopic electric fields. Both electrophoresis and electro-osmosis phenomena need to be considered in micro-channel flow analysis. The fluid pumping that occurs in a micro-capillary, when an electric field is applied along the axis of the capillary is a typical example of the latter. Electro-kinetic transport refers to the combination of electro-osmotic and electro-phoretic transports. A detailed review of electro-kinetic transport is available in a recent publication by Li (2004).

4.1 Electric double layer

The interactions between charged particles and electric fields often involve the electric double layers (EDL), formed at liquid/solid interfaces. The EDL is formed due to the presence of the static charges on the solid surfaces. Generally, a dielectric surface acquires charges when it is in contact with a polar medium or by chemical reaction, ionization or ion absorption. A schematic view of this is shown in Fig. 4. In this figure, Ψ is the electric potential, Ψ_0 the surface electric potential, ζ the zeta potential and y' distance measured from the wall. When the solid surface is in contact with a polar medium, a net electric potential is generated on the surface. Due to this surface electric potential, positive ions in the liquid are attracted to the wall, whereas the negative ions are repelled from the wall. This results in the redistribution of the ions close to the wall, keeping the bulk of the liquid far away from the wall electrically neutral. The distance from the wall, where the electric potential energy is equal to the thermal energy, is known as the Debye length (λ), and this zone is known as the EDL. The electric potential distribution within the fluid is described by the Poisson-Boltzmann equation:

$$\nabla^2(\Psi/\zeta) = \frac{-4\pi h^2 \rho_e}{D\zeta} = \beta \sinh(\alpha\Psi/\zeta)$$

Here α is the ionic energy parameter given as:

$$\alpha = ez\zeta / k_b T$$

Parameter β is as shown below:

$$\beta = (\omega h)^2/\alpha \quad \text{where} \quad \omega = 1/\lambda = \sqrt{\frac{2 n_0 e^2 z^2}{\varepsilon k_b T}}$$

The other symbols in these equations are ρ_e: net charge density, e: electron charge, z: valance, k_b: Boltzmann constant, T: temperature, and h: characteristic length.

4.2 Electro-osmotic flow

Electro-osmosis is the bulk movement of aqueous solution past a stationary solid surface, due to an externally applied electric field. Electro-osmosis requires the existence of a charged double layer at the solid-liquid interface. The electro-osmotic flow is created by applying the electric field in the stream-wise direction, where this electric field ($\mathbf{E}$) interacts with the electric charge distribution (ρ_e) and creates an electro-kinetic body force on the fluid. The ionized incompressible fluid flow with electro-osmotic body forces is governed by the incompressible Navier-Stokes equation:

$$\rho_f\left(\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}.\nabla)\mathbf{V}\right) = -\nabla P + \mu\nabla^2\mathbf{V} + \rho_e\mathbf{E}$$

Here ρ_f, $\mathbf{V}$ and P are the fluid density, velocity vector and pressure, respectively. Based on these continuum based equations, various researchers have developed numerical models to simulate electro-kinetic effects in micro-devices (Yang et al., 1998).

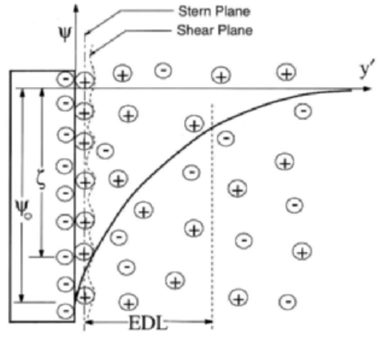

Fig. 4. EDL next to a negatively charged solid surface.

Recently, there is renewed interest in the field of electro-osmotic flow analysis, considering its importance in the micro-channel applications. Qian and Bau (2005) carried out a theoretical investigation of electro-osmotic flows with chaotic stirring in rectangular cavities. Earlier, they (Qian and Bau, 2002) had explained about a new chaotic electro-osmotic stirrer principle. A study about electro-osmatic flow in micro-channels with arbitrary geometry and arbitrary distribution of wall charge was done by Xuan and Li (2005). They have found that the wall charge affects significantly the electro-osmotic flow while the channel geometry does not.

4.3 Electrophoresis

Electrophoresis describes the motion of a charged particle submerged in a fluid under the action of an applied electric field. Considering the case of a charged dye molecule, the electrophoretic velocity, U , of the dye is described by the Helmholtz-Smoluchowski equation (Probstein, 1994):

$$U = \frac{\varepsilon\ \zeta_{EP} E_x}{\mu}$$

The electro-osmotic zeta potential, ζ_{EO} is a pro-perty of the capillary surface while the electrophoretic zeta potential, ζ_{EP} is a property of the charged dye and, in general these two zeta potentials will not be equal.

5. Mixing in micro-channels

5.1 Passive and active micro-mixers

The importance of mixing in micro-channels can never be under-estimated. Efficient micro-mixers are vital for the successful operation of bio-fluid systems. The performance of these devices depends largely on the rapid and efficient mixing of different fluids. This has to be achieved, in spite of the difficulties put forth by the fundamental physics of flow in narrow channels, with high viscosity and low Reynolds number values. Various alternatives are being tried with the aim of reaching the final goal, which is instantaneous mixing (Liu et al., 2000). Also, the effect of fluid mixing needs to be considered in many other micro-systems, like fluid control and pumping systems, where the major aim may be reducing the pressure drop (Koch et al., 2000).

The application of micro-mixers range from the modern sample preparation for the 'lab on a chip' devices to traditional mixing tanks for purposes such as, blending, reaction, gas absorption, foaming and emulsification (Ehrfeld, 2000; Fletcher et al., 2002; Pennemann et al. 2004). The flow rates in micro-mixers range from as low as, less than 1 μl/h to more than 1 ml/h. By-and-large, the micro-flow and mixing is typically done in the laminar regime at very low Reynolds number values (Schonfeld et al., 2004).

In micro-fluidic devices, the mixing relies solely on molecular inter-diffusion (Graveson et al., 1993), due to the absence of turbulence. Diffusive mixing can be optimized by maximization of the constitutive factors like the diffusion coefficient, interfacial surface area and the gradient of species concentration. Basically, 'the art of micro-mixing' translates to an efficient maximization of the interfacial surface area and concentration gradient. Also, convective mixing is commonly employed in the mixing devices.

Mixing in micro-scale is performed either by energy input from the exterior (active mixing) or by the flow energy due to pumping action/hydrostatic potential (passive mixing). Tables 3 and 4 give the various options available in these categories (respectively), and the corresponding literature references.

Table 3. Active mixing by external energy input.

External energy source	Reference
Ultrasound	Yang et al. (2001)
Acoustically induced vibrations	Liu et al. (2003)
Electro-kinetic instabilities	Oddy et al. (2001)
Variation of pumping capacity	Glasgow&Aubry(2003)
Electro-wetting induced droplets	Palk et al. (2003)
Piezo-electric vibrating membranes	Woias et al (2000)
Magneto-hydrodynamic action	West et al. (2002)
Small impellers	Lu et al. (2001)
Integrated micro-valves/pumps	Voldman et al. (1998)

Table 4. Passive mixing by pumping power.

Principle	Reference
Inter-digital multi-lamellae arrangement	Lob et al (2004)
Split-and-recombine concepts	Schonfeld et al. (2004)
Chaotic mixing by eddy formation and folding	Jiang et al. (2004)
Nozzle injection in flow	Miyake et al. (1993)
Collision of jets	Werner et al. (2002)
Specialties like the Coanda-effect	Hong et al. (2001)

Some typical generic micro-structure designs employed for passive mixing includes the following (Lowe et al., 2000).

(i) T and Y flow configurations
(ii) Inter-digital and bifurcation flow distribution
(iii) Focusing structures for flow compression
(iv) Repeated flow division and recombination
(v) Flow obstacles within micro-channels
(vi) Meander-like or zig-zag channels
(vii) Multi-hole plates
(viii) Tiny nozzles
(ix) Specialty flow arrangements

Numerical and experimental investigations on liquid mixing in static micro-mixers were reported by Engler et al. (2004). Their overall aim was to develop basic rules for the design and implementation of successful micro-mixers. Electro-hydrodynamic flow generated by the motion of charge carriers in an electric field was investigated for rapid mixing in micro-channel devices by Tsouris et al. (2003). "T" geometry channels ranging from 40 to 110 μm in width and 30 to 60 μm in depth was used to contact two miscible fluids, such as alcohols in this study. Electro-kinetically driven active micro-mixers utilizing zeta potential variation induced by field effect is investigated by Lee et al. (2004). The development of such electro-kinetically driven micro- mixer could be crucial for μTAS devices. More recently, Sounart et al. (2005) describes about the frequency-dependent electro-static actuation in micro-fluidic MEMS. Their analysis demonstrates the importance of native oxide on silicon actuator response, and suggests that the actuator frequency can be shifted by controlling the thickness of the oxide.

Hessel et al. (2004) presented the vast literature survey on the micro-mixing of miscible fluids in the chemical engineers' point of view.

5.2 Mixing by chaotic advection

The chaotic stirring is a best suited principle for mixing in micro-channels, owing to the existence of low Reynolds number values. Chaotic advection is basically meant to effect stirring in the laminar flow. The salient feature of chaotic advection in the present context is the exponential growth of the interfacial area accompanied by a corresponding reduction of the striation thickness. A comprehensive coverage on the principle of chaotic advection and its mathematical descriptions is given in the book by Ottino (1989). There are many other interesting publications on this topic, available in the literature (for example, Suh, 1991).

Steady, three-dimensional chaotic flows have been successfully used for micro-mixing in several contexts (see Jen, et al., 2003 and Jiang et al., 2004). The first publication on chaotic advection by micro-mixers rely on placing micro-structured objects within the flow passage on one side of the micro-channels (Stroock et al., 2002). A much simpler channel design, based solely on alternatively curved micro-channels, has been proposed by Schonfeld and Hardt (2003). The laminar flow patterns and the mixing performance of the comparable mixers with diagonal or asymmetric grooves was investigated and quantified by Kang and Kwon (2004). More recently, Heo and Suh (2005) have studied the enhancement of stirring in a straight channel at low Reynolds numbers with various block arrangement at different heights and angles. The Lyapunov exponents were computed and it was found that the stirring gets enhanced considerably at larger block heights. Both theoretical and experimental investigations with respect to a magneto- hydrodynamic (MHD) stirrer that exhibits chaotic advection were reported by Yi et al. (2002). Since this device has no moving parts, it is especially suitable for micro-fluidic applications, where efficient stirring is essential.

5.3 Mixing efficiency

The easiest method for judging the mixing efficiency in micro-mixer structures is by flow visualization, which is done using dilution-type experiments (Hassel et al., 2003). Reaction-type experiments underlay the mixing with very fast reaction so that mixed region spontaneously indicate the result of the reaction. Roessler and Rys (2001) introduced two parallel reactions (competitive reactions) which develop differently under varying pH, solvent, etc. This in turn, can be used as a measure of the mixing efficiency. By properly designing the micro-channel manifold, one can accomplish parallel and serial electro-kinetic mixing using a simple voltage-control circuit (Jacobson et al., 1999). Mixing in micro-fluidic devices typically occurs by the interplay of molecular diffusion and pressure-driven convection (dispersion) in the laminar regime. Mixing rates can be enhanced by using the principle of flow lamination (Bessoth et al., 1999), whereby the streams are divided into n laminae with widths $1/n$ of the original channel. This can result in faster mixing by a factor of n^2.

Concentrations are easily accessible by photometric or fluorescence measurements. Besides using photometric techniques, vibrational analysis such as Infrared and Raman can be used following the mixing course in a micro-mixer device (Loebbecke et al., 2002). To quantitatively represent the composition uniformity, Lu et al. (2001) and Bakker et al. (2000) used the coefficient of variation, COV, which is the ratio of the standard deviation of each cross section, σ, to the mean of each cross section, c_{ave}. Wu and Liu (2005) quantitatively characterized the mixing efficiency of their electro-kinetic micro-mixer by defining the mixing index, MI in the same way, as follows:

$$MI = \frac{\sigma}{c_{ave}} = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(\frac{c_i - c_{ave}}{c_{ave}}\right)^2}$$

where N, c_i and c_{ave} are the total number of pixels along line segment, the intensity at pixel i, and the average intensity over N pixels, respectively. The MI value of 1 % or less is considered as a general rule of thumb in the literature for uniform mixing (i.e., at least 99 % complete mixing). This quantity is however often problematic in judging the global mixing performance. For instance, even when a part of the total flow domain is not in a chaotic-mixing regime, the mixing index can show a high value depending on the initial distribution of the concentration (Suh, 2006). We need therefore to develop a robust quantity that represents the mixing performance both reliably and universally.

6. Conclusions

The micro-channel flow analysis is receiving wide attention presently, owing to its high utility potential in many multi-disciplinary applications. Often, it is required to use special numerical techniques for the analysis of flow phenomena in micro-devices. One of the important CFD tools being tried in the recent past, by researchers for the analysis of micro-channel flows includes the Lattice Boltzmann Method (LBM).

The choice of micro-structured mixers is sufficiently broad. However, there are many unresolved issues with respect to micro-mixers which need to be addressed, presently; for instance, development of a more reliable and robust mixing quantity. There is also still room for new designs and understanding principles in this field. More robust and professional designs are needed and field trials have to be conducted to evaluate the potential in an industrial environment. Also benchmarking has to be done and the existing micro-mixers are to be categorized more scientifically.

Acknowledgement

This work was supported by the Korea Science and Engineering Foundation(KOSEF) through the National Research Lab. Program funded by the Ministry of Science and Technology (No. 2005-01091). This paper was also partially supported by the Dong-A University Research Fund in 2006. SJ wishes to acknowledge the support being received from the National Institute of Technology Calicut, India. We especially thank one of the reviewers for his providing us with the book written by Hessel et al. (2004) as an additional reference in micro-mixing.

References

Aluru, N., 1999, “A Reproducing Kernel Particle Method for Meshless Analysis,” *Computational Mech.,* Vol. 23, pp. 324~338.

Angelopoulos, A. D., Paunov, V. N., Burganos, V. N., Payatakes, A. C., 1998, “Lattice Boltzmann Si-mulation of Non-Ideal Vapor-Liquid Flow in Porous Media,” *Phys. Rev E 57.,* Vol. 3, pp. 3237~3245.

Arkilic, E. B., Schmidt, M. A., Breuer, K. S., 1997, “Gaseous Flow in Long Microchannel,” *J. MEMS,* Vol. 6, pp. 167~178.

Bakker, A., Richard, D. L., Elizabeth, M. M., 2000, “Laminar Flow in Static Mixers with Helical Elements,” *The Online CFM Book,* http://www.Bakker.Org/cfm-book/ pp. 1~11.

Bessoth, F. G., deMello, A. J., Manz, A., 1999, “Microstructure for Efficient Continuous Flow Mixing,” *Anal. Commun.*, Vol. 36, p. 213~215.

Bird, G. A., 1998, “Recent Advances and Current Challenges for DSMC,” *Computers Math. Applic.,* Vol. 35, No. 1, pp. 1~14.

Branebjerg, J., Gravesen, P., Krog, J. P., Neilsen, C. R., 1996, “Fast Mixing by Lamination,” *Proc. 9th Ann. Workshop of Micro Electro Mech. Sys.*, pp. 441~446.

Brody, J. P., Yager, P., Goldstein, R. E., Austin, R. H., 1996, "Biotechnology at Low Reynolds Numbers," *Biophys. J.,* Vol. 71, pp. 3430~3441.

Chen, S., Doolen, G. D., 1998, "Lattice Boltzmann Method for Fluid Flows," *Ann. Rev. Fluid Mech.,* Vol. 30, pp. 329~364.

Choi, S. B., Barron, R. F., Warrington, R. O., 1991, "Fluid Flow and Heat Transfer in Microtubes," in DSC-*Vol. 32, Micromechanical Sensors, Actuators and Systems,* ASME Winter Annual Meeting, Atlanta, GA, pp. 123~134.

Cummings, E. B., Griffiths, S. K., Nilson, R. H., 1999, "Irrotationality of Uniform Electroosmosis," in *SPIE Conf. on Microfluidic Devices and Systems II,* Sanata Clara, CA, Vol. 3877, pp. 180~189.

Ehrfeld, W., Hessel, V., Lowe, H., 2000, *Microreactors*, Wiley-VCH, Weinheim.

Engler, M., Kockmann, N, Kiefer, T., Woias, P., 2004, "Numerical and Experimental Investigations on Liquid Mixing in Static Micromixers," *Chem. Eng.,* Vol. 101, pp. 315~322.

Fletcher, P. D. I., Haswell, S. J., Pombo-Villar, E., Warrington, B. H., Watts, P., Wong, S. Y. F., 2002, "Micro Reactors: Principle and Applications in Organic Synthesis," Tetrahedron, Vol. 58, No. 24, pp. 4735~4757.

Flockhart, S. M., Dhariwal, R. S., 1998, "Experimental and Numerical Investigations into the Flow Characteristics of Channels Etched in <100> Silicon," *Fluids Eng.,* Vol. 120, pp. 291~295.

Freeman, D., Aranyosi, A., Gordon, M., Hong. S., 1998, "Multidimensional Motion Analysis of MEMS Using Computer Microvision," in *Solid-State Sensor and Actuator Workshop,* Hilton Head Island, S. C., pp. 150~155.

Frisch, U., Hasslacher, B., Pomeau, Y., 1986, "Lattice-Gas Automata for the Navier-Stokes Equation," *Phys. Rev. Lett.,* Vol. 56, No. 14, pp. 1505~1508.

Gad-el-Hak, M. (ed), 2006, *The MEMS Handbook,* Second Edition, Tayler & Francis, Boca Raton.

Gen, G., 2001, "Phonon Heat Conduction in Low-Dimensional Structures," *Semiconductors & Semimetals,* Vol 71, p. 203.

Glasgow, I., Aubry, N., 2003, "Enhancement of Microfluidic Mixing using Time Pulsing," *Lab on a Chip*, Vol. 3, pp. 114~120.

Graveson, P., Branjeberg, J., Jensen, O.S., 1993, "Microfluidics – A Review," *J. Micromech., Microeng.,* Vol. 3, No.4, pp. 168~182.

Hassel, E., Jahnke, S., Kornev, N., Tkatchenko, I., Zhdanov, V., 2003, "Laminar Mixing in Different Interdigital Micromixers–Part I: Experimental Characterization," *A.I.Ch.E. J.*, Vol. 49, No.3, pp. 566~577.

Hessel, V., Lowe, H., Muller, A., Kolb, G., 2004, *Chemical Micro Process Engineering, Process and Plants*, Wiley-VCH.

Heo, H. S., Suh, Y. K., 2005, "Enhancement of Stirring in a Straight Channel at Low Reynolds-Numbers with Various Block-Arrangement," *KSME Int. J. Mech. Sci.,* Vol. 19, No. 1, pp. 199~208.

Ho, C. -M., Tai, Y. -C., 1998, "Microelectromechanical Systems (MEMS) and Fluid Flows," *Ann. Rev. Fluid Mech.*, Vol. 30, pp. 579~612.

Hong, C. C., Choi, J. W., Ahn, C. H., 2001, "A Novel In-plane Passive Micromixer Using Coanda Effect," in van den Berg, A., Olthuis, W., Bergveld, P., (Eds.), *Micro Total Analysis Systems,* Kluwer Academic Publishers, Dordrecht, pp. 31~33.

Jacobson, S. C., McKnight, T. E., Ramsey, J. M., 1999, "Microfluidic Devices for Electrokinetically Driven Parallel and Serial Mixing," *Anal. Chem.*, Vol. 71, pp. 4455~4459.

Jen, C. P., Wu, C. Y., Lin, Y. C., Wu, C. Y., 2003, "Design and Simulation of the Micromixer with Chaotic Advection in Twisted Microchannels," *Lab on a Chip*, Vol. 3, pp. 77~81.

Jiang, X. N., Zhou, Z. Y., Yao, J., Li., Y., Ye, X. Y., 1995, "Micro-Fluid Flow in Microchannel," in *Transducers'95: Eurosensor IX, 8th Int. Conf. on Solid-State Sensors and Actuators and Eurosensors IX,* Sweden, pp. 317~320.

Jiang, F., Drese, K. S., Hardt, S. Kupper, M., Schonfeld, 2004, "Helical Flows and Chaotic Mixing in Curved Micro Channels," *A.I.Ch.E. J.,* Vol. 50, No. 9, pp. 2297~2305.

Kang, T. G., Kwon, T. H., 2004, "Colored Particle Tracking Method for Mixing Analysis of Chaotic Micro- mixers," *J. Micromech. & Microeng.,* Vol. 14, pp. 891~899.

Knight, J., 1999, "Dust Mite's Dilemma," *New Scientist,* Vol. 162, No. 2180, pp. 40~43.

Koch, M., Evans, A., Brunnschweiler, A., 2000, *Microfluidic Technology and Applications*, Research Studies Press Ltd.

Lee, C. Y., Lee, G. B., Fu, L. M., Lee, K. H., Yang, R.J., 2004, "Electrokinetically Driven Active Micromixers Utilizing Zeta Potential Variation Induced by Field Effect," *Micromech. Microeng.,* Vol. 14, pp. 1390~1398.

Li, D., 2004, *Electrokinetics in Microfluidics,* Elsevier Academic Press, Amsterdam.

Li, Z., Kang J., Park J.H., Suh, Y.K., 2007, "Numerical Simulation of the Droplet Formation in a Cross-Junction Microchannel Using the Lattice Boltzmann Method," *KSME Int. J. Mech. Sci.,* Vol. 21, No. 1, pp. 162~173.

Liu, R. H., Stremler, M. A., Sharp, K. V., Olsen, M. G., Santaigo, J. G., Adrian, R. J., Aref, H., Beebe, D. J., 2000 "Passive Mixing in a Three-Dimensional Serpantine Microchannel," *J. MEMS,* Vol. 9, pp. 190~197.

Liu, R. H., Lenigk, R., Druyor-Sanchez, R. L., Yang, J., Grodzinski, P., 2003, "Hybridization Enhancement using Cavitation Microstreaming," *Analytical Chemistry*, Vol. 75, No. 8, pp. 1911~1917.

Lob, P., Pennemann, H., Hassel, V., 2004, "Steering of Liquid Mixing speed in Interdigital Micromixers – from Very Fast to Deliberately Slow Mixing," *Chem. Eng. Tech.*, Vol. 27, No. 3, pp. 340~345.

Loebbecke, S., Antes, J., Tuercke, T., Boskovich, D., Schweikert, W., Marioth, E., Schnuerer, F., Krause, H.H., 2002, "Black Box Microreactor? Possibilities for a Better Control and Understanding of Microreaction Processes by Applying Suitable Analytics," In: *6th Int. Conf. on Microreaction Tech., IMRET 6,* New Orleans, USA, A.I.Ch.E. Publication No. 164, pp. 37~38.

Lowe, H., Wille, Ch., Guber, A. J., 2000, "Micromixing Technology," in *4th Int. Conf. on Microreaction Technology, IMRET 4,* Atlanta, USA, A.I.Ch.E. Topical Conf. Proc., pp. 31~47.

Lu, L. H., Ryu, K. S., Liu, C., (Eds.), 2001, "A Novel Microstirrer and Arrays for Microfluidic Mixing," in van den Berg, A., Olthuis, W., Bergveld, P., (Eds.), *Micro Total Analysis Systems,* Kluwer Academic Publishers, Dordrecht, pp. 28~30.

Mala, G.M., Li, D., 1999, "Flow Characteristics of Water in Microtubes," *Int. J. Heat Fluid Flow,* Vol. 20, pp. 142~148.

Miyake, R., Lammerink, T. S. J., Elwenspoek, M., Fluitaman, J. H. J., 1993, "Micromixer with Fast Diffusion," in *Proc. IEEE-MEMS'93*, Fort Lauderdale, FL, pp. 248~253.

Oddy, M. H., Santiago, J. G., Mikkelsen, J. C., 2001, "Electrokinetic Instability Micromixing," *Anal. Chem.,* Vol. 73, No. 24, pp. 5822~5832.

Oran, E. S., Oh, C. K., Cybyk, B. Z., 1998, "Direct Simulation Monte Carlo: Recent Advances and Applications," *Ann. Rev. Fluid Mech.,* Vol. 30, pp. 403~441.

Ottino, J. M., 1989, *The Kinematics of Mixing: Stretching, Chaos, and Transport,* Cambridge Univ. Press, Oxford.

Palk, P., Pamula, V. K., Fair, R. B., 2003, "Rapid Droplet Mixers for Digital Microfluidic Systems," *Lab on a Chip*, Vol.3, pp. 253~259.

Papautsky, I., Brazzle, J., Ameel, T. A., Frazier, A. B., 1999a, "Laminar Fluid Behavior in Microchannels Using Micropolar Fluid Theory," *Sensors and Actuators A,* Vol. 73, pp. 101~108.

Papautsky, I., Gale, B. K., Mohanty, S, Ameel, T. A., Frazier, A. B., 1999b, "Effect of Rectangular Microchannel Aspect Ratio on Laminar Friction Constant," in *Proc. SPIE Conf. on Microfluidic Devices and Systems II,* Santa Clara, CA, Vol. 3877, pp. 147~158.

Peng, X. F., Peterson, G. P., Wang, B. X., 1994, "Friction Flow Characteristics of Water Flowing Through Rectangular Microchannels," *Exp. Heat Transfer,* Vol. 7, pp. 249~264.

Pennemann, H., Hassel, V., Lowe, H., 2004, "Chemical Micro Process Technology–From Laboratory Scale to Production," *Chem. Eng. Sci.*, Vol. 59, No. 22~23, pp. 4789~4794.

Pfahler, J., Harley, J., Bau, H., Zemel, J.N., 1991, " Gas and Liquid Flow in Small Channels," in *DSC Vol. 32, Micromechanical Sensors, Actuators and Systems,* ASME Winter Annual Meeting, Atlanta, GA, pp. 49~59.

Probstein, R. F., 1994, *Physiochemical Hydrodynamics: An Introduction,* 2nd Edn., Wiley and Sons Inc., New York.

Qian, S., Bau, H. H., 2002, "A Chaotic Electroosmatic Stirrer," *Anal. Chem.,* Vol. 74, No. 15, pp. 3616~ 3625.

Qian, S., Bau, H. H., 2005, "Theoretical Investigation of Electro-Osmatic Flows and Chaotic Stirring in Rectangular Cavities," *Applied Mathematical Modeling,* Vol. 29, pp. 726~753.

Qian, Y. H., D'Humieres, D., Lallemand, P., 1992, "Lattice BGK Models for Navier-Stokes Equation," *Europhys. Lett.,* Vol. 17, pp. 479~482.

Qu, W., Mala, G. M., Li, D., 2000, "Pressure-Driven Water Flows in Trapizoidal Silicon Microchannels," *Int. J. Heat Mass Transfer,* Vol. 43, pp. 353~364.

Rasmussen, A., Zaghloul, M. E., 1999, "The Design and Fabrication of Microfluidic Flow Sensors," in *Proc. ISCAS-99,* pp. 136~139.

Roessler, A., Rys. P., 2001, "Selektivitat mischungsmaskierter reaktionen: Wenn die Ruhrgesch- windigkeit die Produktverteilung bestimmt," *Chemie in unsurer Zeit,* Vol. 35, No. 5, pp. 314~322.

Santiago, J. G., Wereley, S. T., Meinhart, C. D., Beebe, D. J., Adrian, R. J., 1998, "A Particle Image Velocimetry System for Microfluidics," *Exp. Fluids,* Vol. 25, pp. 316~319.

Schonfeld, F., Hardt, S., 2003, "Simulation of Helical Flows in Microchannels," *A.I. Ch.E. J.,* Vol. 50, No. 4, pp. 2297~2305.

Schonfeld, F., Hessel, V., Hofmann, C., 2004, "An Optimized Split-and-Recombine Micro Mixer with Uniform 'Chaotic' Mixing," *Lab on a Chip,* Vol. 4, pp. 65~69.

Schulte, T. M., Bardell, R. L., Weigl, B. H., 2000, "On-Chip Microfluidic Sample Preparation," *J. Lab. Autom.*, Vol. 5, p. 83.

Senturia, S., Aluru, N., White, J., 1997, "Simulating the Behavior of MEMS Devices: Computational Methods and Needs," *IEEE Computational Sci. Eng.,* January-March, pp. 30~43.

Sharp, K. V., Adrian, R. J., Beebe, D. J., 2000, "Anomalous Transition of Turbulence in Microtubes," in *Proc. Int. Mech. Eng. Cong. Expo., 5th Micro-Fluidic Symp.,* Orlando, FL.

Sounart, T. L., Michalske, T. A., Zavadil, K. R., 2005, "Frequency-Dependent Electrostatic Actuation in Micro-fluidic MEMS," *Microelectromech. Sys.* Vol. 14, No. 1, pp. 125~133.

Stroock, A. D., Dertinger, S. K. W., Ajdari, A., Mezic, I., Stone, H. A., Whitesides, G. M., 2002, "Chaotic Mixing in Micro- channels," *Science,* Vol. 295, No. 1, pp. 647~651.

Suh, Y. K., 1991, "A Chaotic Stirring by an Oscillating Point Vortex," *Phy. Soc. Japan,* Vol. 60, No. 3, pp. 896~906.

Suh, Y. K., 2006, "On the Problem of Using Mixing Index Based on the Concentration Dispersion," *Trans. Of the KSME(B)*, Vol. 30, No. 8, pp. 796~805.

Sukop, M. C., Thorne, Jr. D. T., 2006, *Lattice Boltzmann Modeling: An Introduction for Geoscientists and Engineers,* Springer-Verlag.

Tieu, A. K., Mackenzie, M. R., Li, E. B., 1995, "Measurements in Microscopic Flow with Solid-State LDA," *Exp. Fluids,* Vol. 19, pp. 293~294.

Tsouris, C, Culbertson, C. T., DePaoli, D. W., Jacobson, S. C., de Almeida, V. F., Ramsey, J. M., 2003, "Electro-hydrodynamic Mixing in Microchannels," *A. I. Ch. E.*, Vol. 49, No. 8, pp. 2181~2186.

Tuckerman, D. B., Pease, R. F. W., 1981, "High-Performance Heat Sinking with VLSI," *IEEE Electron Device Lett.,* EDL-2, pp. 126~129.

Voldman, J., Gray, M. L., Schmidt, M. A., 1998, "Liquid Mixing Studies with an Integrated Mixer/ Valve," In: van den Berg, A., Olthuis, W., Bergveld, P., (Eds.), *Micro Total Analysis Systems,* Kluwer Academic Publishers, Dordrecht, pp. 181~184.

Werner, B., Donnet, M., Hessel, V., Hofmann, C., Jongen, N., Lowe, H., Schenk, 2002, "Specially Suited Micromixers for Process Involving Strong Fouling," in *Proc. 6th Int. Conf. on Microreaction Technology, IMRET 6*, New Orleans, USA, A.I.Ch.E. Publication No. 164, pp. 168~183.

Webb, A and Maynes, D., 1999, "Velocity Profile Measurements in Microtubes," in *30th AIAA Fluid Dyn. Conf.,* Norfolk, VA, AIAA 99-3803.

Weigl, B. H., Yager, P., 1999, "Microfluidic Diffusion-Based Separation and Detection," *Science,* Vol. 292, pp. 346~347.

West, J., Karamata, B., Lillis, B., Gleeson, J. P., Alderman, J. Collins, J. K., Lane, W., Mathewson, A., Berney, H., 2002, "Application of Magneto- hydrodynamic Actuation to Continuous Flow Chemistry," *Lab on a Chip,* Vol. 2, pp. 224~230.

Wilding, P., Pfahler, J., Bau, H. H., Zemel, J. N., Kricka, L. J., 1994, "Manipulation and Flow of Biological Fluids in Straight Channels Micromachined in Silicon," *Clin. Chem.*, Vol. 40, pp. 43~47.

Woias, P., Hauser, K., Yacoub-George, E., (Eds.), 2000, "An Active Silicon Micromixer for μTAS Applications," in van den Berg, A., Olthuis, W., Bergveld, P., (Eds.), *Micro Total Analysis Systems*, Kluwer Academic Publishers, Dordrecht, pp. 277~282.

Wu, H.Y., Liu, C.H., 2005, "A Novel Electrokinetic Micromixer," *Sensors and Actuators A,* Vol. 118, pp. 107~115.

Wu, P., Little, W. A., 1983, "Measurement of Friction Factors for the Flow of Gases in Very Fine Channels Used for Microminiature Joule-Thomson Refrigerents," *Cryogenics*, Vol. 23, pp. 273~277.

Xuan, X, Li, D, 2005, "Electroosmatic Flow in Microchannels with Arbitrary Geometry and Arbitrary Distribution of Wall Charge," *Colloid and Interface Sci.,* Vol. 289, pp. 291~303.

Yang, C., Li, D., Masliyah, J. H., 1998, "Modeling Forced Liquid Convection in Rectangular Microchannels with Electrokinetic Effects," *Int. J. Heat Mass Transfer*, Vol. 41, pp. 4229~4249.

Yang, Z., Matsumoto, S. Goto, H, Matsumoto, M and Maeda, R, 2001, "Ultrasonic Micromixer for Microfluidic Systems," *Sensors and Actuators A*, Vol. 93, pp. 266~272.

Ye, W., Kanapka, J., White, J., 1999, "A Fast 3D Solver for Unsteady Stokes Flow with Applications to Micro-Electro-Mechanical Systems" in *Proc. Second Int. Conf. on Modeling and Simulation of Microsystems,* pp. 518~521.

Yi, M., Qian, S., Bau, H. H., 2002, "A Magneto-hydrodynamic Chaotic Stirrer," *Fluid Mech.,* Vol. 468, pp. 153~177.

Yu, D., Warrington, R., Barron, R., Ameel, T., 1995, "An Experimental and Theoretical Investigation of Fluid Flow and Heat Transfer in Microtubes," in *Proc. ASME/ JSME Thermal Engineering Joint Conf.,* Maui, HI, pp. 523~530.